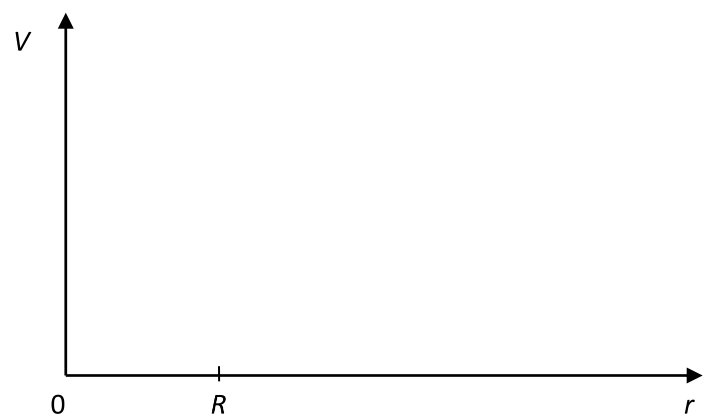
Problem of the week

Electric fields (HL)

(a) A conducting hollow charged sphere of radius R has a charge Q on its surface.

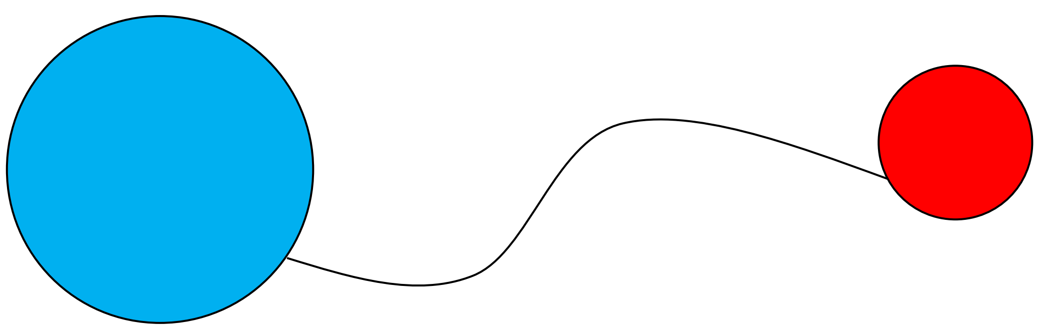
(i) State what is meant by *electric potential* at a point.

(ii) Sketch, on the axes, a graph to show the variation of the electric potential V with distance r from the center of the sphere.



(iii) The hollow sphere is replaced by solid sphere of the same radius and charge. Suggest how the graph in (ii) changes, if at all.

(b) A conducting sphere of radius R has charge Q on its surface. The sphere is joined with a long conducting wire to a smaller conducting sphere of radius $\frac{R}{2}$ which is initially uncharged.



(i) State and explain why the surfaces of the two spheres must now be at the same electric potential.

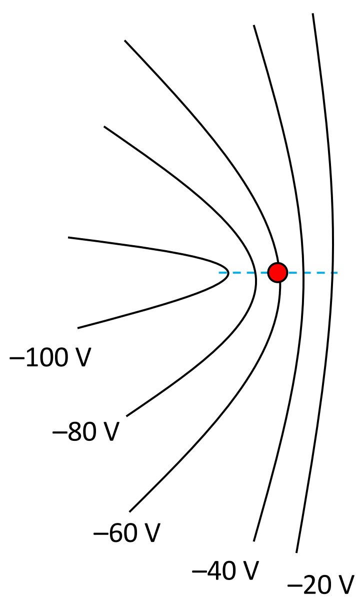
(ii) Determine, in terms of Q, the charge on each sphere.

(iii) Surface charge density is defined as charge per unit area. Before the connection was made the surface charge density of the large sphere was σ. Determine, in terms of σ, the surface charge density of each sphere after the connection is made.

(iv) Show that the electric field strength on the surface of a conducting sphere is $E = \frac{\sigma}{\varepsilon_0}$ where σ is the surface charge density.

(v) Hence, or otherwise, determine the ratio $\frac{E_R}{E_{\frac{R}{2}}}$ of the electric field strength on the surfaces of the two spheres.

(c) The diagram shows five equipotential lines (cross-sections of equipotential surfaces on a the plane of the page). A particle of charge – 12 μC is placed on the – 60 V line as shown. Along the blue dotted line, the equipotential lines are separated by 4.0 cm.



(i) State and explain what can be deduced about the magnitude of the electric field along the blue dotted line.

(ii) Draw an arrow to indicate the direction of the electric field at the position of the particle.

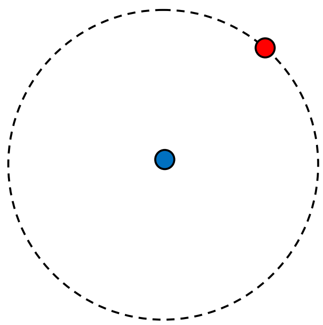
(iii) Estimate the electric force on the particle.

(iv) State the direction of motion of the particle when it is released.

(v) Determine the change in the electric potential energy of the particle when it reaches the next equipotential line.

(vi) Calculate the work done by the electric force for the motion in (v).

(d) An electron is in a circular orbit around a proton. The orbit radius is 0.53×10^{-10} m.



(i) Calculate the period of revolution of the electron.

(ii) Show that the total energy of the electron is given by $E_T = -\frac{ke^2}{2r}$.

(iii) Evaluate, in eV, this energy for the orbit radius of 0.53×10^{-10} m.

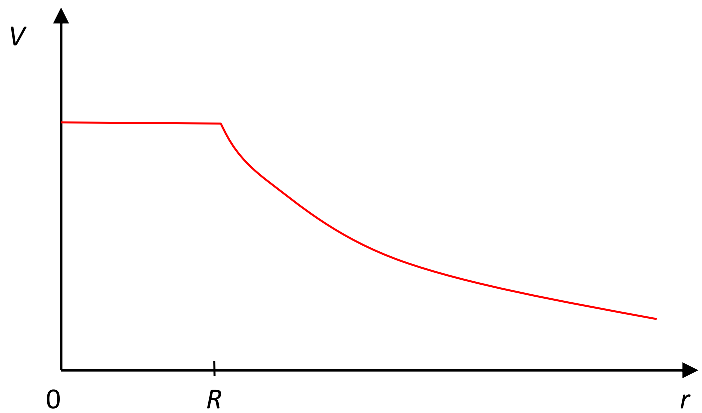
(iv) The electron radiates electromagnetic waves. Explain why the electron's orbit radius will decrease.

Answers

(a)

(i) The work done per unit charge by an external agent in bringing a point positive charge from infinity to the point at an infinitesimally small, constant speed.

(ii)



(iii) No change at all.

(b)

(i) Charge will move from one sphere to the other. The motion of the charges will stop when the potential difference becomes zero.

(ii) $\frac{Q_1}{R} = \frac{Q_2}{\frac{R}{2}} \Rightarrow Q_2 = \frac{Q_1}{2}$. But $Q_1 + Q_2 = Q$ (conservation of charge) and so

$Q_1 + \frac{Q_1}{2} = Q \Rightarrow Q_1 = \frac{2Q}{3}$ and $Q_2 = \frac{Q}{3}$.

(iii) $\sigma_1 = \frac{\frac{2Q}{3}}{4\pi R^2} = \frac{2}{3}\frac{Q}{4\pi R^2} = \frac{2}{3}\sigma$.

$\sigma_2 = \frac{\frac{Q}{3}}{4\pi \frac{R^2}{4}} = \frac{4}{3}\frac{Q}{4\pi R^2} = \frac{4}{3}\sigma$.

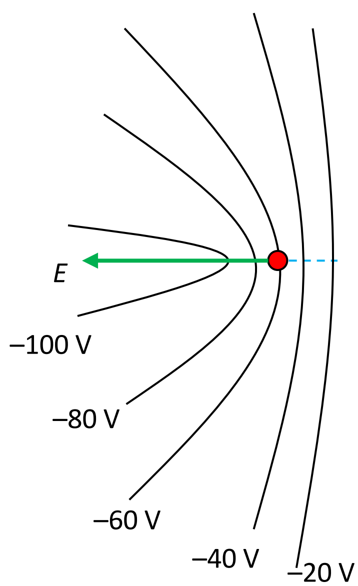
(iv) $E = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{\sigma(4\pi R^2)}{4\pi\varepsilon_0 R^2} = \frac{\sigma}{\varepsilon_0}$.

(v) $\frac{E_R}{E_{\frac{R}{2}}}=\frac{\sigma_1}{\sigma_2}=\frac{1}{2}$ ***OR*** $\frac{E_R}{E_{\frac{R}{2}}}=\frac{\frac{kQ_1}{R^2}}{\frac{kQ_2}{(\frac{R}{2})^2}}=\frac{1}{4}\times\frac{\frac{2Q}{3}}{\frac{Q}{3}}=\frac{1}{2}$

(c)

(i) The distances between the equipotential lines are the same and so is the potential difference hence from $E=\frac{\Delta V}{\Delta x}$ the magnitude of the electric field strength is constant along the dotted line.

(ii) The electric field is normal to the equipotential lines in the direction of decreasing potential:



(iii) The electric field strength has magnitude $E=\frac{\Delta V}{\Delta x}=\frac{20}{4.0\times 10^{-2}}=500\text{ N C}^{-1}$. The electric force is then $F=qE=12\times 10^{-6}\times 500=6.0\text{ mN}$.

(iv) It will move opposite the electric field so to the right.

(v) $\Delta E_P=qV_{\text{final}}-qV_{\text{initial}}=(-12\times 10^{-6})\times(-40)-(-12\times 10^{-6})\times(-60)=-2.4\times 10^{-4}\text{ J}$.

(vi) $W=Fs=6.0\times 10^{-3}\times 4.0\times 10^{-2}=2.4\times 10^{-4}\text{ J}$ (since the force is constant)

OR

$W_{\text{field}}=-q\Delta V=(-12\times 10^{-6})\times(-40-(-60))=-2.4\times 10^{-4}\text{ J}$.

(d)

(i) $\frac{ke^2}{r^2}=\frac{mv^2}{r}\Rightarrow v^2=\frac{ke^2}{mr}$ and so

$v=\frac{2\pi r}{T}\Rightarrow \frac{4\pi^2 r^2}{T^2}=\frac{ke^2}{mr}$

$T=\sqrt{\frac{4\pi^2 mr^3}{ke^2}}=1.5\times 10^{-16}$ s

(ii) $E_T=\frac{1}{2}mv^2-\frac{ke^2}{r}=\frac{ke^2}{2r}-\frac{ke^2}{r}=-\frac{ke^2}{2r}$.

(iii) $E_T=-\frac{8.99\times 10^9\times(1.6\times 10^{-19})^2}{2\times 0.53\times 10^{-10}}=2.17\times 10^{-18}$ J. Hence

$E_T=-\frac{2.17\times 10^{-18}}{1.6\times 10^{-19}}=-13.6\approx -14$ eV.

(iv) The EM waves carry away energy decreasing the total energy of the electron, i.e., it becomes more negative. This implies that the radius will decrease.